

THE SPECTRAL GAP WHEN POWER LEAKS INTO MORE THAN ONE TYPE OF SURFACE WAVE ON PRINTED-CIRCUIT LINES

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ABSTRACT

A new type of spectral gap has been found to exist at the onset of leakage into more than one type of surface wave. The properties of this type of spectral gap have been investigated theoretically and experimentally for conductor-backed coplanar strips.

INTRODUCTION

It is by now well known that the dominant mode on most printed-circuit transmission lines will become leaky above some critical frequency. The power leaks at an angle from the transmission line in the form of a surface wave on the surrounding substrate. It is also understood now that at the transition between the bound and leaky regions there exists a small range in frequency within which the solution is *nonphysical*. This transition range is called a *spectral gap*.

The details of the spectral gap's fine structure can be calculated very accurately, and its nature is now understood rather well. Examples of this type of spectral gap have appeared in many recent papers [1-3]. In the bound region, the field of the dominant mode *decays* exponentially in the transverse direction, whereas the field *increases* exponentially transversely in the leaky region. In the transition between the bound and leaky regions, where the field must change from transverse decay to transverse increase, the field must at some point have no variation transversely. This limit point is clearly nonphysical because it corresponds to zero energy density for the field. Within the spectral gap we also find a frequency range that contains a pair of nonphysical improper real solutions. An example of a spectral gap [4] is shown in Fig. 1 which illustrates some of the points mentioned above.

At lower frequencies only a single surface wave, usually the TM_0 surface wave, is above cutoff (and, in fact, propagates down to zero frequency). When the frequency increases to the value for which k_s , the wavenumber of the surface wave, becomes greater than the value of β , the wavenumber of the dominant mode on the printed-circuit line, the dominant mode changes from bound to leaky, and the transition corresponds to the spectral gap. As indicated above, this spectral gap is now well understood. When the frequency is increased further, the next surface wave, usually the TE_1 surface wave, can also be above cutoff. Furthermore, when the wavenumber for that surface wave becomes greater than β , leakage will occur into *both* of the surface waves above cutoff, that is, into both the TE_1 and the TM_0 surface waves, although at different angles.

For convenience, let us call the wavenumbers for the first and second surface waves (that is, the TM_0 and the TE_1 surface waves, respectively) k_{s1} and k_{s2} . When $k_{s1} > \beta > k_{s2}$, leakage occurs into the TM_0 surface wave only, but when both k_{s1} and k_{s2} are greater than β , the leaky mode leaks power into both surface waves. As k_{s2} becomes about equal to β , therefore, *another* spectral gap is encountered. At the

transition from bound to leaky, where only the TM_0 mode was involved, we had a real solution on one side of the spectral gap and a complex solution on the other. Here, we have a complex solution on *both* sides, making the nature of this *new* spectral gap *different* and unclear.

In this paper, we explore the properties of this new spectral gap both theoretically and experimentally. To our knowledge, this is the first time that this new spectral gap has been identified, and any of its properties examined.

CALCULATIONS USING THE SPECTRAL-DOMAIN METHOD

The spectral-domain approach requires an integration in the k_x plane, where k_x is the wavenumber in the transverse direction parallel to the surface of the substrate. When the dominant mode is purely bound, the integration is performed along the real axis. When leakage occurs into only a single surface wave, the integration path is deformed to also include the pole corresponding to the surface wave into which the dominant mode leaks. When leakage occurs into two surface waves, the integration path includes both of the corresponding poles. This procedure is considered standard. The surface-wave poles are then located in the first quadrant of the k_x plane; when the leakage occurs only into the first surface wave, the corresponding pole lies in the first quadrant, but the pole for the second surface wave (and all the other surface

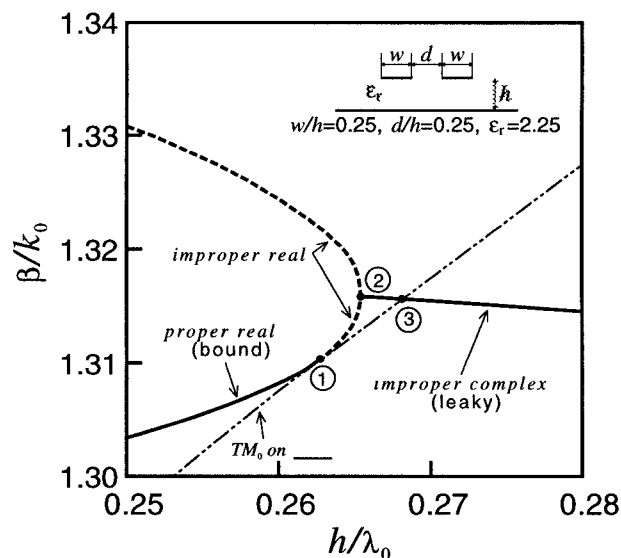


Fig. 1. An example of the usual type of spectral gap, which holds for the structure shown in the inset. In this expanded plot the spectral gap extends between points ① and ③. In the frequency range between points ① and ② we find a pair of nonphysical improper real solutions.

waves, whether above or below cutoff), lies in the third quadrant. The sketches in Figs. 2(a) and (b) illustrate the paths required in the k_x plane when leakage occurs into only one surface wave or two surface waves, respectively.

Using the above approach, calculations were made of the longitudinal wavenumber behavior as a function of frequency for a specific printed-circuit transmission line. The structure chosen was a pair of conductor-backed coplanar strips, with the geometrical parameters shown in the inset appearing in Fig. 3(a). The normalized strip widths are $w/h = 0.583$, the normalized spacing between the strips is $d/h = 0.243$, $\epsilon_r = 2.25$, and h is the substrate height. A recent paper [5] showed that, when the relative dimensions of various types of printed-circuit lines are modified, the spectral gap at the first onset of leakage can be of the usual type or can disappear and be replaced by a simultaneous-propagation effect. We chose conductor-backed coplanar strips for this calculation, and also selected dimensions for operation under simultaneous-propagation conditions, because the leakage constant then becomes rather large and the spectral gap between the cases for leakage into one surface wave and into two surface waves becomes more pronounced. For this structure, the first surface wave is the TM_0 surface wave and the second one is the TE_1 surface wave.

The numerical results obtained using the spectral-domain method for the propagation wavenumber k_z , where $k_z = \beta - j\alpha$, are shown in Figs. 3(a) and 3(b) for β/k_0 vs. h/λ_0 (normalized frequency) and α/k_0 vs. h/λ_0 , respectively. Also shown in these figures are the first three surface waves that are capable of propagating on the surrounding substrate in this frequency range. At the lower frequencies, the solid line represents the bound mode; the β values are seen to be greater than those for the TM_0 surface wave (and, of course, those for all the other surface waves). The other solid line, which lies below the curve for the TM_0 surface wave, is the solution corresponding to leakage into the TM_0 surface wave only, obtained by deforming the integration path to enclose only the TM_0 pole in the k_x plane. (It may be seen that the two solutions

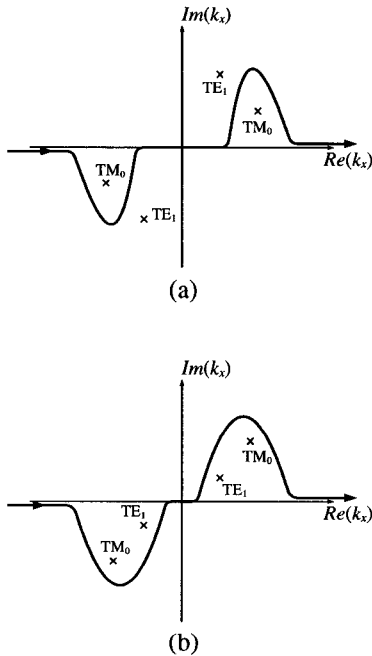


Fig. 2. Integration paths in the k_x plane for the spectral-domain approach when leakage occurs into (a) one surface wave, and (b) two surface waves.

represented by solid lines exist simultaneously over a certain frequency range, corresponding to what was referred to above as a range of simultaneous propagation. These solutions also do not end abruptly as they appear to here; they connect smoothly to nonphysical solutions, which are not shown here but which are discussed in detail in [5][6].)

A third guided-mode solution, which is shown by large dashes and which lies above the leaky solid-line solution, corresponds to leakage into both the TM_0 and TE_1 surface waves. It was obtained by deforming the path according to Fig. 2(b). For convenience, we shall call the leaky solid-line solution LM_1 and the leaky dashed-line solution LM_2 , where LM stands for "leaky mode" and the subscript indicates the number of surface waves into which power leaks.

The change from leakage into the TM_0 surface wave only and leakage into both the TM_0 and TE_1 surface waves is expected to occur in the neighborhood of $\beta = k_{s2}$, where k_{s2} is the wavenumber of the TE_1 surface wave. But we see in Fig. 3(a) that the LM_1 and LM_2 solutions cross the k_{s2} curve in **different** places, and that in fact there is a **jump** between the two solutions. Solution LM_1 is expected to be valid at frequencies **below** f_{s1} in Fig. 3(a), and solution LM_2 is expected to be valid **above** f_{s2} . It is unclear what to trust in the range between f_{s1} and f_{s2} . Solutions LM_1 and LM_2 are also shown in Fig. 3(b) for α/k_0 , and the same statements apply there.

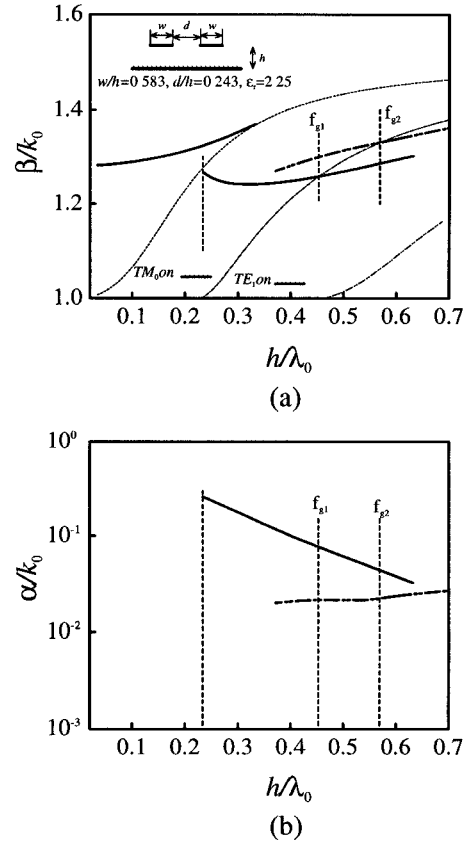


Fig. 3. Theoretical values, obtained by means of the spectral-domain method, of β/k_0 and α/k_0 as a function of normalized frequency, h/λ_0 , for conductor-backed coplanar strips. The structure and relative dimensions are shown in the inset of (a). Frequencies f_{s1} and f_{s2} represent the edges of the spectral gap, and solutions LM_1 and LM_2 are the two solutions that overlap in the spectral gap, with a jump between them. See the discussion in Sec. II for further explanation.

The fact that two separate solutions arise, and that there is a jump between them, has been observed previously for other structures, and has been described in a paper [7] in the context of possible alternative solutions and their physical meaning. No conclusions were drawn there, however, except to suggest that each solution gradually loses physical meaning when they overlap each other.

We shall present measured results in the next section that provide evidence that a spectral gap of some type exists between frequencies f_{g1} and f_{g2} . This is the first example of any measurements of this type, and we believe that this paper is the first that clearly identifies this transition region as an actual spectral gap that contains a nonphysical solution, rather than simply as a transition that permits a gradual transfer of physical meaning from one solution to another.

MEASURED RESULTS

We constructed a transmission line consisting of conductor-backed coplanar strips with the same relative dimensions as those for which we made the calculations shown in Figs. 3(a) and 3(b). The actual dimensions were $h = 10.3$ mm, $d = 2.5$ mm, and $w = 6.0$ mm. The onset and end frequencies (f_{g1} and f_{g2}) of the spectral gap, which correspond to $h/\lambda_0 = 0.456$ and 0.576 , respectively, are actually 13.28 and 16.78 GHz.

The measurement procedures required vary with the range of the h/λ_0 values. For the bound mode, the phase constant was measured using the standing-wave pattern produced by a strongly reflecting termination. For the leaky modes, the phase constants were obtained in the simultaneous-propagation region from leakage-angle measurements. For frequencies higher than that region, the phase constants of the leaky modes were found from phase-shift measurements taken by a vector network analyzer. The leakage constants of the leaky modes were obtained by making field-intensity measurements along the guide axis (the z direction), with special considerations required in the simultaneous-propagation region.

Measurements *within* the spectral gap (between f_{g1} and f_{g2}) were particularly difficult to make. Repeated measurements showed considerable scatter, but a clear pattern emerged. The data which we are reporting represent a careful average of the scattered points in that region. Another region for which the measurements were found to be hard to make is at the onset of leakage, where solution LM_1 crosses the TM_0 surface wave curve. In this vicinity, the leakage angle is very small, with the leakage direction almost parallel to the guide axis.

The results of these measurements are presented in Figs. 4(a) and 4(b), as a set of round open dots superimposed on the theoretical values already presented in Figs. 3(a) and 3(b). Despite some scatter, the agreement with the theoretical values is actually quite good.

For α/k_0 in the neighborhood of the onset of leakage, the measured values drift away and down from the theoretical curve, and this trend seems to be clear despite the difficulty mentioned above for this region. This behavior is physically satisfying because the theoretical curve connects with a nonphysical complex solution when $\beta = k_{s1}$.

What is truly new here is the behavior *within* the spectral gap, between f_{g1} and f_{g2} , both for β/k_0 and for α/k_0 . It is the first time, to our knowledge, that measurements of any kind have been made for this type of spectral gap, and it is gratifying that a clear pattern seems to have emerged.

The pattern found for the β/k_0 measurements is that the

values first follow solution LM_1 at the lower frequency end, near f_{g1} , then move up to solution LM_2 and follow that curve to frequency f_{g2} and beyond it. The pattern for the α/k_0 measurements is that soon after f_{g1} the α/k_0 values drop very significantly from the LM_1 solution; there is then a region of extremely low values, after which the values increase rapidly and then (with some scatter) tend to follow the LM_2 solution.

The next question is how these patterns should be interpreted.

DISCUSSION OF THE RESULTS

From the theoretical calculations presented in Fig. 3, we know that in the frequency range from the onset of leakage to frequency f_{g1} the leaky mode leaks power into the TM_0 surface wave only, and that for frequencies greater than f_{g2} the leaky mode leaks into both the TM_0 and TE_1 surface waves. The theoretical solutions valid for these two separate frequency ranges are called LM_1 and LM_2 , respectively. If we extend solutions LM_1 and LM_2 through the region between f_{g1} and f_{g2} and slightly beyond, as we have done in Fig. 3, and then hope that their ranges of validity can likewise be extended, we find that the two solutions do not meet but that there is a jump between them instead. We are therefore led to the conclusion that both solutions cannot simultaneously be valid in that region, and that some unusual behavior must be occurring there.

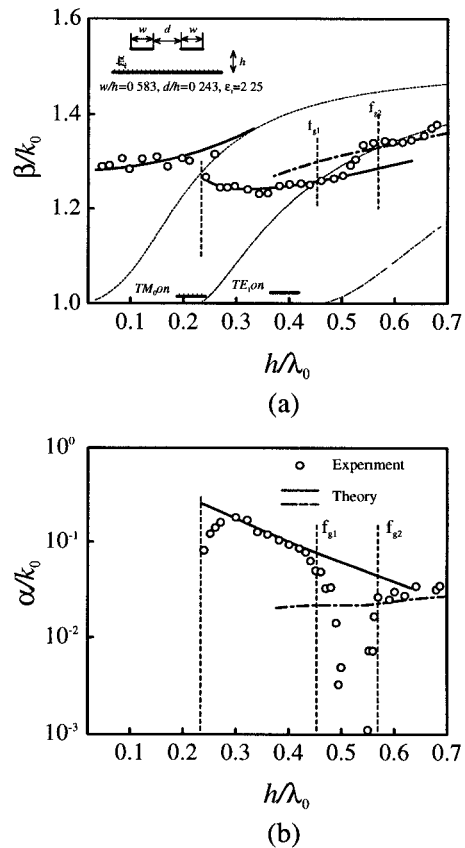


Fig. 4. Measured results of β/k_0 and α/k_0 for conductor-backed coplanar strips. The structure is shown in the inset, and the actual dimensions are $h = 10.3$ mm, $d = 2.5$ mm, and $w = 6.0$ mm. The measured values are presented at round dots, which are superimposed on the theoretical curves already appearing in Figs. 3(a) and 3(b). Measured values have actually been taken in the spectral gap region (between f_{g1} and f_{g2}), and they form patterns which are described in Sec. III and discussed in Sec. IV.

In an attempt to obtain additional insight into the behavior within the region between f_{g1} and f_{g2} , we took a series of measurements. A comparison between these measured results and the theoretical calculations are shown in Fig. 4. As discussed there, measured values were actually obtained in the region between f_{g1} and f_{g2} , as well as outside of it. Outside of this region, the agreement between the measured and theoretical values is quite good, but within that region the behaviors are significantly different. In fact, the measured values exhibit a *distinct pattern of behavior* for both β/k_0 and α/k_0 . The pattern for β/k_0 shows that the measured values first follow the LM_1 or LM_2 solutions as we enter this region from the lower-frequency or the higher-frequency ends, respectively, and that they then join each other in the middle of the region. The measurements for α/k_0 show that soon after entering the region from either end the α/k_0 values drop very significantly. The central portion of this region is seen to have extremely low values of attenuation.

Before attempting to understand what exactly gives rise to this behavior between f_{g1} and f_{g2} , we must recognize that the eigenvalue solutions LM_1 and LM_2 do not explain this behavior, and that therefore it is completely appropriate to call the region between f_{g1} and f_{g2} a *spectral gap*.

In agreement with information about the behavior of other spectral gaps, we should expect that the spectral gap boundaries are not sharp, with the LM_1 and LM_2 solutions completely valid outside and invalid inside. Instead, the validity should be expected to extend somewhat inside of the spectral gap, with a gradual loss of validity as one penetrates further into the spectral gap. This would explain the behavior near the gap's edges, where the measured values follow the theoretical values for LM_1 and LM_2 . What is unclear, and requires further interpretation, is the behavior near the middle of the spectral gap. One must explain why the β/k_0 values join each other, and why the α/k_0 values drop so strongly.

It is reasonable to assume that, with respect to the eigenvalue solutions, there is a nonphysical region in the middle portion of the spectral gap. If the eigenvalue solution there is nonphysical, any measured values for β/k_0 or α/k_0 would not result from the eigenvalue solution but from some radiation in surface-wave or space-wave form from the exciting source. From the nature of the excitor, the most likely candidate would be TE_1 surface-wave radiation which would spread out from the excitor. The measured values of β would then be equal to k_{s2} , the wavenumber of the TE_1 surface wave. If we look at Fig. 4(a), for β/k_0 , we see that when $\beta = k_{s2}$ we are directly on the curve for the TE_1 surface wave. That would immediately explain why the LM_1 and LM_2 solutions seem to connect with each other.

The explanation for the drop in the α/k_0 values is then the following. Outside of the spectral gap, but especially at the low-frequency end, the value of the leakage constant is very high, so the field decay is very rapid. When the eigenvalue solution becomes nonphysical, and only the spreading surface wave is being measured, the decay rate (which is not exponential so that a quantitative comparison cannot be made in general terms) becomes very much reduced.

The above reasoning for both the β/k_0 and the α/k_0 behaviors is therefore consistent with the interpretation that a nonphysical region exists in the middle portion of the spectral gap.

A further interpretation offers an additional argument for the strong drop in the values of α/k_0 . This argument is based on the recognition that for the LM_1 solution, which corresponds to the integration path in Fig. 2(a), the TE_1 pole associated with the TM_0 pole in the first quadrant actually lies in

the third quadrant, because both are related to propagation in the $+x$ direction. Similarly, the TE_1 pole in the first quadrant is actually paired with the TM_0 pole in the third quadrant, corresponding to propagation in the $-x$ direction. For the LM_2 solution, for which one uses the integration path in Fig. 2(b), the TE_1 and TM_0 poles are paired in the same quadrant because both are now leaking power. To go from solution LM_1 to solution LM_2 , therefore, the TE_1 pole must somehow migrate from the third quadrant to the first quadrant if we are considering leakage into the $+x$ direction, for example. (The two solutions need not connect directly to each other.) Such a migration requires that the TE_1 pole must cross the horizontal axis in the k_x plane at some point.

Then, let us write

$$k_z = \beta - j\alpha \quad \text{and} \quad k_x = \beta_x - j\alpha_x. \quad (1)$$

We also recognize that

$$k_{s2}^2 = k_z^2 + k_x^2. \quad (2)$$

Substituting (1) into (2) and taking the imaginary part, we obtain

$$2\alpha\beta = -2\alpha_x\beta_x. \quad (3)$$

When the TE_1 pole crosses the horizontal axis in the k_x plane, we have $\alpha_x = 0$. From (3) we see that $\alpha\beta$ must then be zero; since β is not zero, α must equal zero when the TE_1 pole crosses the horizontal axis. We therefore have another explanation for the strong drop in the α/k_0 values, and a result that is consistent with the pattern of the measured results.

We are thus led to the conclusion that there is actually a spectral gap between f_{g1} and f_{g2} , and that the central portion of this gap contains a nonphysical guided-wave region.

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